PARITY-VIOLATING ANOMALIES IN SUPERSYMMETRIC GAUGE THEORIES

E.R. NISSIMOV and S.J. PACHEVA

Institute of Nuclear Research and Nuclear Energy, Boulevard Lenin 72, 1784 Sofia, Bulgaria

Received 28 January 1985

The parity-violating part of the one-loop effective action of scalar superfields in parity-invariant three-dimensional supersymmetric gauge theories is explicitly found by means of the superspace heat kernel method Its interpretation as a superspace analogue of the Atiyah–Patodi–Singer η-invariant is proposed

1. Parity-violating anomalies (PVA) in gauge theories with massless fermions in odd space–time dimensions D (we shall consider euclidean space–time below) were recently widely discussed [1–8], first, because they are natural analogues of the usual chiral anomalies in even D, and secondly, due to their close relationship to some interesting physical phenomena such as fermion number fractionization [3,6] and the quantized Hall effect [6,7]. Conditions for occurrence or avoiding of PVA versus dynamical spontaneous breakdown of parity and the dependence of PVA on the asymptotic behaviour of the gauge field strength at space–time infinity were analyzed in ref. [8]. The most concise way of expressing PVA is the following formula first proposed by Polyakov [4] (cf. also ref. [5]) for the logarithm of the determinant of the Dirac operator $\nabla\!\!\!\!/(A)$ (N_f being the number of fermion flavors):

$$N_\mathrm{f}\ln\det[-\mathrm{i}\nabla\!\!\!\!/(A)] = N_\mathrm{f}\tfrac{1}{2}\ln\det[\nabla\!\!\!\!/^2(A)] - N_\mathrm{f}\tfrac{1}{2}\mathrm{i}\pi\eta_{\nabla\!\!\!\!/}[A] - N_\mathrm{f}S_\mathrm{ct}[A], \tag{1}$$

$$\ln\det[\nabla\!\!\!\!/^2(A)] = \int_0^\infty \mathrm{d}\tau\,\tau^{-1}\mathrm{Tr_R}[\exp\{-\tau\nabla\!\!\!\!/^2(A)\}], \quad \eta_{\nabla\!\!\!\!/}[A] = \int_0^\infty \mathrm{d}\tau\,(\pi\tau)^{-1/2}\mathrm{Tr_R}[\nabla\!\!\!\!/(A)\exp\{-\tau\nabla\!\!\!\!/^2(A)\}]. \tag{2,3}$$

Definition (1) is formally valid for arbitrary self-adjoint operators. $\mathrm{Tr_R}[\ ]$ means the (infrared) regularized operator trace defined by subtraction of the corresponding operator at $A_\mu = 0$. In (1)–(3) and below the following notations are used:

$$\{\gamma_\mu,\gamma_\nu\} = -2\delta_{\mu\nu}, \quad \gamma_\mu^+ = -\gamma_\mu, \quad \nabla\!\!\!\!/(A) = \gamma_\mu\nabla_\mu(A) = \gamma_\mu[\partial_\mu + \mathrm{i}A_\mu(x)],$$

$$A_\mu = T^a A_\mu^a, \qquad \{T^a\} \quad (a = 0,1,\ldots,n^2-1) \quad \text{hermitian generators of } \mathrm{U}(n),$$

$$A_\mu(x) = -\mathrm{i}g^{-1}(x_\infty)(\partial_\mu g)(x_\infty) + \mathrm{O}(|x|^{-1-\epsilon}) \quad \text{for } |x|\to\infty,\ x_\infty\in S_\infty^{D-1}.$$

In (1), (3) $\eta_{\nabla\!\!\!\!/}[A]$ denotes the well-known spectral asymmetry measuring η-invariant [9] of $\nabla\!\!\!\!/(A)$, where

$$\eta_{\nabla\!\!\!\!/}[A] = (-1)^{(D+1)/2}W_\mathrm{ChS}^{(D)}[A] + B[A], \quad \eta_{\nabla\!\!\!\!/}[A^g] = \eta_{\nabla\!\!\!\!/}[A], \quad \eta_{\nabla\!\!\!\!/}[A^\mathrm{P}] = -\eta_{\nabla\!\!\!\!/}[A], \tag{4}$$

under gauge- and parity-transformations

$$A_\mu^g(x) = g^{-1}(x)[A_\mu(x) - \mathrm{i}\partial_\mu]g(x), \quad \psi^g(x) = g^{-1}(x)\psi(x), \quad g(x)\in \mathrm{G} = \mathrm{U}(n), \tag{5}$$

$$(A^\mathrm{P})_\mu(x) = (A_0, -A_1, A_2, \ldots, A_{D-1})(x^\mathrm{P}), \quad \psi^\mathrm{P}(x) = -\mathrm{i}\gamma_1\psi(x^\mathrm{P}), \quad x^\mathrm{P} \equiv (x^0, -x^1, x^2, \ldots, x^{D-1}). \tag{6}$$

The appearance of $\eta_{\not\nabla}[A]$ in (1) represents the general form of PVA unless (a) N_f = even or/and (b) the homotopy group $\pi_D(\mathrm{U}(n)) \neq \mathrm{Z}$, the group of integers (which is true for $D > 2n$, see e.g. ref. [10]. In these latter cases (a), (b) one can choose the counterterm $S_{\mathrm{ct}}[A]$ (local functional of A_μ accounting for the renormalization ambiguity) in the form $S_{\mathrm{ct}} = \mathrm{i}\pi(-1)^{(D-1)/2} W^{(D)}_{\mathrm{ChS}}[A]$ such that PVAs in (1) are avoided. In (4) $W^{(D)}_{\mathrm{ChS}}[A]$ denotes the well-known odd-dimensional Chern–Simons secondary class [11], in particular, in $D = 3$:

$$W^{(3)}_{\mathrm{ChS}}[A] = -(16\pi^2)^{-1}\epsilon_{\mu\nu\lambda}\int \mathrm{d}^3x\, \mathrm{tr}[A_\mu F_{\nu\lambda}(A) - \mathrm{i}\tfrac{2}{3}A_\mu A_\nu A_\lambda], \tag{7}$$

$$W^{(D)}_{\mathrm{ChS}}[A^g] = W^{(D)}_{\mathrm{ChS}}[A] + n_D[g], \quad W^{(D)}_{\mathrm{ChS}}[A^{\mathrm{P}}] = -W^{(D)}_{\mathrm{ChS}}[A], \tag{8}$$

$$n_{D=3}[g] = -(24\pi^2)^{-1}\epsilon_{\mu\nu\lambda}\int \mathrm{d}^3x\, \mathrm{tr}[(g^{-1}\partial_\mu g)(g^{-1}\partial_\nu g)(g^{-1}\partial_\lambda g)], \tag{9}$$

where $n_D[g]$ (element of $\pi_D(\mathrm{U}(n)) = \mathrm{Z}$ for all odd $D < 2n$, [10]) is the topological charge of $g(x)$. $B[A]$ in (4) denotes a piece-wise constant functional of A_μ, whose values are even integers:

$$\delta B[A]/\delta A^a_\mu(x) = 0 \quad \text{for all } A_\mu(x) \text{ whose } \not\nabla(A) \text{ do not possess zero modes,}$$

$$B[A^g] = B[A] + (-1)^{(D-1)/2}2n_D[g], \quad B[A^{\mathrm{P}}] = -B[A]. \tag{10}$$

In fact, $B[A]$ may be associated to twice the index of an appropriate $D + 1$ = even-dimensional Dirac operator [9].

The aim of the present note is to find generalizations of (1)–(4), (9), (10) to the case of $D = 3$ supersymmetric gauge theories.

2. The (euclidean) superspace action of $D = 3$ (massive) scalar superfields $\Phi(x, \theta)$ in an external (not quantized) gagge superfield $\mathcal{A}_\alpha(x, \theta)$ reads [12]:

$$S[\Phi, \mathcal{A}] = \int \mathrm{d}^3x\, \mathrm{d}^2\theta\, \Phi^*[\nabla^2(\mathcal{A}) - \mathrm{i}m]\Phi = \int \mathrm{d}^3x\, D^2(\Phi^*[\nabla^2(\mathcal{A}) - \mathrm{i}m]\Phi)|_{\theta=0} \tag{11}$$

$$= \int \mathrm{d}^3x\, \{\varphi^*[\tfrac{1}{2}\nabla_{\alpha\beta}(A)\nabla^{\alpha\beta}(A) + m^2]\varphi + \mathrm{i}\bar\psi^\alpha[\nabla_\alpha{}^\beta(A) - m\delta_\alpha{}^\beta]\psi_\beta + \tfrac{1}{2}\mathrm{i}\varphi^*\lambda^\alpha\psi_\alpha - \tfrac{1}{2}\mathrm{i}\bar\psi^\alpha\lambda_\alpha\varphi\}, \tag{11'}$$

$$\Phi^{(*)}(x, \theta) = \varphi^{(*)}(x) + \theta^\alpha \overset{(-)}{\psi}_\alpha(x) + \delta(\theta)F^{(*)}(x), \quad \mathcal{A}_\alpha(x,\theta) = \chi_\alpha(x) + \mathrm{i}A_{\alpha\beta}(x)\theta^\beta + b(x)\theta_\alpha + \delta(\theta)\lambda_\alpha(x) = \mathcal{A}_\alpha{}^a(x,\theta)T^a. \tag{11'',''' }$$

The component-field action (11′) is written in the Wess–Zumino gauge [$\chi_\alpha = b = 0$ in (11‴)] and after eliminating the auxiliary fields $F^{(*)}$. Also, the standard spinor- and superspace notations are used:

$$D_\alpha = \partial/\partial\theta^\alpha + \mathrm{i}\theta^\beta\partial_{\alpha\beta}, \quad \nabla_\alpha(\mathcal{A}) = D_\alpha + \mathrm{i}\mathcal{A}_\alpha, \quad \alpha = 1,2, \qquad \nabla_{\alpha\beta}(A) = \partial_{\alpha\beta} + \mathrm{i}A_{\alpha\beta},$$

$$X^2 \equiv \tfrac{1}{2}X^\alpha X_\alpha, \quad X^\alpha = C^{\alpha\beta}X_\beta, \quad X^\alpha{}_\beta \equiv -\mathrm{i}(X_\mu\gamma_\mu)^\alpha{}_\beta, \quad C_{\alpha\beta} = -C^{\alpha\beta} = (\sigma_2)_{\alpha\beta}, \quad \gamma_{1,2} = \mathrm{i}\sigma_{1,2}, \quad \gamma_0 = \mathrm{i}\sigma_3,$$

$$\delta(\theta) = -\tfrac{1}{2}\theta^{(\alpha)}\theta_\alpha.$$

The action (11) is invariant under superfield gauge transformations:

$$\Phi^\omega(x,\theta) = \omega^+(x,\theta)\Phi(x,\theta), \quad \mathcal{A}_\alpha{}^\omega(x,\theta) = \omega^+(x,\theta)[\mathcal{A}_\alpha(x,\theta) - \mathrm{i}D_\alpha]\omega(x,\theta),$$

$$\omega^{(+)}(x,\theta) = g^{(+)}(x) + \theta^\alpha \overset{(-)}{\zeta}_\alpha(x) + \delta(\theta)S^{(+)}(x), \quad \omega^+(x,\theta) = \omega^{-1}(x,\theta), \quad \omega(x,\theta) \in \mathrm{U}(n), \tag{12}$$

and under parity-transformation for $m = 0$:

$$\Phi^P(x,\theta) = \Phi(x^P,\theta^P), \quad \mathcal{A}^{P,\alpha}(x,\theta) = (\sigma_1)^\alpha{}_\beta \mathcal{A}^\beta(x^P,\theta^P), \quad \theta^{P,\alpha} \equiv (\sigma_1)^\alpha{}_\beta \theta^\beta. \tag{13}$$

The mass term $-\mathrm{i}m\int \mathrm{d}^2\theta\ \Phi^*\Phi$ changes sign under (13).

As in the usual theories, the heat kernel representation of superfield propagators (for $D = 4$ superspace, see ref. [13]) will prove very useful. Kernels of superspace differential operators $\mathcal{P}(\nabla_\alpha)$, $\mathcal{P}(\ )$ being an arbitrary polynomial, are defined as $\mathcal{P}(\nabla_\alpha)(x,\theta;x',\theta') = \mathcal{P}(\nabla_\alpha)\delta^{(3)}(x-x')\delta(\theta-\theta')$. In particular, let us introduce the operator $\nabla^4 \equiv [\nabla^2(\mathcal{A})]^2$

$$\nabla^4 = -[\partial_\mu + \mathrm{i}\Gamma_\mu(\mathcal{A})][\partial_\mu + \mathrm{i}\Gamma_\mu(\mathcal{A})] + \mathrm{i}W^\alpha(\mathcal{A})\nabla_\alpha(\mathcal{A}),$$

$$\Gamma^\alpha{}_\beta(\mathcal{A}) = -\mathrm{i}[\Gamma_\mu(\mathcal{A})\gamma_\mu]^\alpha{}_\beta = -\tfrac{1}{2}\mathrm{i}[D^\alpha\mathcal{A}_\beta + D_\beta\mathcal{A}^\alpha + \mathrm{i}\{\mathcal{A}^\alpha,\mathcal{A}_\beta\}],$$

$$W_\alpha(\mathcal{A}) = \tfrac{1}{2}D^\beta D_\alpha\mathcal{A}_\beta + \tfrac{1}{2}\mathrm{i}[\mathcal{A}^\beta, D_\beta\mathcal{A}_\alpha] - \tfrac{1}{6}[\mathcal{A}^\beta,\{\mathcal{A}_\beta,\mathcal{A}_\alpha\}],$$

the latter being the super gauge field strength. From the "heat" equation $(\partial/\partial\tau)\mathcal{K} = \nabla^4\mathcal{K}$ the following asymptotic expansion for the heat kernel $\mathcal{K} = \exp(-\tau\nabla^4)$ of ∇^4 may be deduced:

$$\exp(-\tau\nabla^4)(x,\theta;x',\theta') = \sum_{l=0}^{\infty} \tau^{(l-4)/4}(2\pi)^{-4}\int \mathrm{d}^3\xi\ \exp\{\mathrm{i}\tau^{-1/2}\xi(x-x')\}$$

$$\times \int_\Gamma \mathrm{i}\,\mathrm{d}\lambda\ \mathrm{e}^{-\lambda} R_{-3-l/2}(x,\theta;\lambda,\xi,\widetilde{D})\delta(\tau^{-1/4}(\theta-\theta')). \tag{14}$$

Unless explicitly stated, we shall assume that $\nabla^4 \equiv \nabla^4(\mathcal{A})$ does not possess "zero modes", i.e. that ∇^4 is invertible. The integration contour Γ in (14) is shown in fig. 1. $R_{-3-l/2}$ are determined recursively:

$$\sum_{2|L|+k+l=p=0,1,2,\ldots} (L!)^{-1}\int \mathrm{d}^2\theta''\ \partial_\xi^L \sigma_{1-k/2}(x,\theta;\lambda,\xi,\widetilde{D})\delta(\theta-\theta'')$$

$$\times (-\mathrm{i}\partial_x)^L R_{-3-l/2}(x,\theta'';\lambda,\xi,\widetilde{D})\delta(\theta''-\theta') = \delta_{0p}\delta(\theta-\theta'), \tag{15}$$

$L = (L_1,\ldots,L_k)$ (multiindex), $L! = L_1!\ldots L_k!$, in terms of the superspace symbol σ of the operator $\nabla^4 - \lambda$:

$$[\nabla^4 - \lambda]\delta^{(3)}(x-x')\delta(\theta-\theta') = (2\pi)^{-3}\int \mathrm{d}^3\xi\ \mathrm{e}^{\mathrm{i}\xi(x-x')}\sigma(x,\theta;\lambda,\xi,\widetilde{D})\delta(\theta-\theta'),$$

$$\sigma(x,\theta;\lambda,\xi,\widetilde{D}) = \sum_{l=0}^{4}\sigma_{1-l/2}(x,\theta;\lambda,\xi,\widetilde{D}),$$

$$\sigma_{1-l/2}(x,\theta;\rho^2\lambda,\rho\xi,\widetilde{D})\delta(\rho^{-1/2}(\theta-\theta')) = \rho^{1-l/2}\sigma_{1-l/2}(x,\theta;\lambda,\xi,\widetilde{D})\delta(\theta-\theta'), \quad \rho > 0, \tag{16}$$

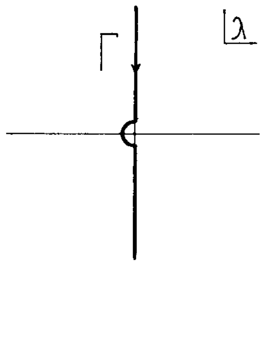

Fig. 1. Integration contour in (14), (25)

$$\sigma_1 = \xi^2 - \lambda, \quad \sigma_{1/2} = 0, \quad \sigma_0 = 2\xi_\mu \Gamma_\mu(\mathcal{A}), \quad \sigma_{-1/2} = \mathrm{i} W^\alpha(x,\theta)\widetilde{D}_\alpha,$$

$$\sigma_{-1} = -\mathrm{i}\,\partial_\mu \Gamma_\mu(\mathcal{A}) + \Gamma_\mu(\mathcal{A})\Gamma_\mu(\mathcal{A}) - W^\alpha \mathcal{A}_\alpha . \tag{16 cont'd}$$

In (14)–(16) $\widetilde{D}_\alpha = \partial/\partial\theta^\alpha - \theta^\beta \xi_{\alpha\beta}$, where $\xi^\alpha{}_\beta = -\mathrm{i}(\xi_\mu \gamma_\mu)^\alpha{}_\beta$, acts only on the Grassmann δ-functions. $R_{-3-l/2}$ are rational functions of λ, ξ and polynomials in $\widetilde{D}_\alpha$ of at most second degree (due to $\{D_\alpha, D_\beta\} = 2\mathrm{i}\partial_{\alpha\beta}$). $R_{-3-l/2}$ satisfy the homogeneity relations ($\rho > 0$):

$$R_{-3-l/2}(x,\theta;\rho^2\lambda,\rho\xi,\widetilde{D})\delta(\rho^{-1/2}(\theta-\theta')) = \rho^{-3-l/2} R_{-3-l/2}(x,\theta;\lambda,\xi,\widetilde{D})\delta(\theta-\theta'). \tag{17}$$

The formal sum

$$R(x,\theta;\lambda,\xi,\widetilde{D})\delta(\theta-\theta') = \sum_{l=0}^{\infty} R_{-3-l/2}(x,\theta;\lambda,\xi,\widetilde{D})\delta(\theta-\theta') \tag{18}$$

may be considered as a superspace symbol for the resolvent $[\nabla^4 - \lambda]^{-1}$. Let us note the complete similarity of (14)–(18) to the formulas for the ordinary heat kernel expansions within the symbol calculus of pseudodifferential operators (e.g. ref. [14]).

3. The one-loop effective action for massless $\Phi(x,\theta)$,

$$\exp(-S_{\mathrm{eff}}[\mathcal{A}]) = \int \mathcal{D}\Phi\,\mathcal{D}\Phi^* \exp(-S[\Phi,\mathcal{A}]) = \exp(-\mathrm{sTr}_{\mathrm{R}} \ln[\nabla^2(\mathcal{A})]), \tag{19}$$

must be ultravioletly regularized so as to preserve invariance under (12). A standard choice is the Pauli–Villars regularization,

$$(\mathrm{sTr}_{\mathrm{R}} \ln[\nabla^2(\mathcal{A})])^{\mathrm{ren}} = \lim_{M\to\infty} \{\mathrm{sTr}_{\mathrm{R}} \ln[\nabla^2(\mathcal{A})] - \mathrm{sTr}_{\mathrm{R}} \ln[\nabla^2(\mathcal{A}) - \mathrm{i}M]\} + S_{\mathrm{ct}}[\mathcal{A}], \tag{20}$$

where $S_{\mathrm{ct}}[\mathcal{A}]$ (a local functional of $\mathcal{A}_\alpha$) is a counterterm accounting for the renormalization ambiguity. Clearly, $S_{\mathrm{ct}}[\mathcal{A}]$ should be invariant under (12) but it need not be parity-invariant, since parity (13) is explicitly broken by Pauli–Villars regularization.

It is simpler instead of (20) to analyse the renormalized expression for the induced supercurrent:

$$J^{(a)}_\alpha(x,\theta) = \mathrm{i}\langle \Phi^* T^a \nabla_\alpha \Phi - (\Phi \overleftarrow{\nabla}_\alpha)^* T^a \Phi \rangle = 2[\delta/\delta\mathcal{A}^{\alpha,a}(x,\theta)]\,\mathrm{sTr}_{\mathrm{R}} \ln[\nabla^2(\mathcal{A})]. \tag{21}$$

Inserting (20) into (21) and using the heat kernel representation we get:

$$[J^a_\alpha(x,\theta)]^{\mathrm{ren}} = \lim_{M\to\infty} 2\mathrm{i}\,\mathrm{tr}\{T^a \nabla_\alpha \nabla^2[(\nabla^4)^{-1} - (\nabla^4 + M^2)^{-1}](x,\theta;x,\theta)\}$$

$$- \lim_{M\to\infty} 2M\,\mathrm{tr}[T^a \nabla_\alpha (\nabla^4 + M^2)^{-1}(x,\theta;x,\theta)] + 2[\delta/\delta\mathcal{A}^{\alpha,a}(x,\theta)]S_{\mathrm{ct}}[\mathcal{A}] \tag{22}$$

$$= \lim_{M\to\infty} 2\mathrm{i} \int_0^\infty \mathrm{d}\tau (1 - \mathrm{e}^{-\tau M^2})\,\mathrm{tr}[T^a(\nabla_\alpha \nabla^2 \mathrm{e}^{-\tau\nabla^4})(x,\theta;x,\theta)]$$

$$- \lim_{M\to\infty} 2M \int_0^\infty \mathrm{d}\tau\,\mathrm{e}^{-\tau M^2}\,\mathrm{tr}[T^a(\nabla_\alpha \mathrm{e}^{-\tau\nabla^4})(x,\theta;x,\theta)] + 2[\delta/\delta\mathcal{A}^{\alpha,a}]S_{\mathrm{ct}}[\mathcal{A}]. \tag{23}$$

Now one can easily check by means of (14)–(16) that no ultraviolet divergences (i.e. singularities of the form $O(\tau^{-k})$, $k \geqslant 1$) appear in the first term in (23) when the regularization is removed. Also, this term is parity-normal [cf. (13)]:

$$[J^a_\alpha(x,\theta)]^{(\mathrm{normal})} = 2\mathrm{i} \int_0^\infty \mathrm{d}\tau\, \mathrm{tr}[T^a(\nabla_\alpha \nabla^2 \mathrm{e}^{-\tau\nabla^4})(x,\theta;x,\theta)] = [\delta/\delta \mathcal{A}^{\alpha,a}(x,\theta)]\,\mathrm{sTr_R} \ln[\nabla^4], \tag{24}$$

whereas the second term in (23) is parity-anomalous. For the latter, accounting for (14)–(16), we get:

$$\begin{aligned}[J^a_\alpha(x,\theta)]^{(\mathrm{PVA})} &= -\lim_{M\to\infty} 2M^{-1} \int_0^\infty \mathrm{d}\rho\, \mathrm{e}^{-\rho}\, \mathrm{tr}[T^a(\nabla_\alpha \mathrm{e}^{-(\rho/M^2)\nabla^4})(x,\theta;x,\theta)] \\ &= -\lim_{M\to\infty} 2 \int_0^\infty \mathrm{d}\rho\, \rho^{-1/2} \mathrm{e}^{-\rho}\Big((2\pi)^{-4} \int \mathrm{d}^3\xi \int_\Gamma \mathrm{i}\,\mathrm{d}\lambda\, \mathrm{e}^{-\lambda}\, \mathrm{tr}[T^a \widetilde{D}_\alpha R_{-3-3/2}(x,\theta;\lambda,\xi,\widetilde{D})\delta(\theta-\theta')]_{\theta=\theta'} \\ &\quad + \mathrm{O}((\rho/M^2)^{5/2})\,(s \geqslant 1)\Big) \\ &= -\mathrm{i}(4\pi)^{-1}\, \mathrm{tr}[T^a W_\alpha(x,\theta)]\,. \end{aligned} \tag{25}$$

Note that $J^{a(\mathrm{PVA})}_\alpha$ may also be represented in the form

$$[J^a_\alpha(x,\theta)]^{(\mathrm{PVA})} = \mathrm{i}\pi[\delta/\delta \mathcal{A}^{\alpha,a}(x,\theta)]\,\eta^{\mathrm{SUSY}}_{\nabla^2}[\mathcal{A}], \quad \eta^{\mathrm{SUSY}}_{\nabla^2}[\mathcal{A}] \equiv \int_0^\infty \mathrm{d}\tau\, (\pi\tau)^{-1/2}\, \mathrm{sTr_R}[(-\nabla^2)\mathrm{e}^{-\tau\nabla^4}], \tag{26}$$

which follows from the operator identity

$\delta(\mathrm{Tr}[P \exp(-t^2P^2)]) = (\mathrm{d}/\mathrm{d}t)(t\, \mathrm{Tr}[(\delta P)\exp(-t^2P^2)])$.

From (26) and (25) we obtain:

$$\eta^{\mathrm{SUSY}}_{\nabla^2}[\mathcal{A}] = 2W^{\mathrm{SUSY}}_{\mathrm{ChS}}[\mathcal{A}] + \mathcal{B}[\mathcal{A}], \tag{27}$$

$$\begin{aligned} W^{\mathrm{SUSY}}_{\mathrm{ChS}}[\mathcal{A}] &= -(16\pi^2)^{-1} \int \mathrm{d}^3x\, \mathrm{d}^2\theta\, \mathrm{tr}\{\mathcal{A}^\alpha W_\alpha(\mathcal{A}) + \tfrac{1}{6}\, \mathcal{A}^\alpha[\mathcal{A}^\beta, \Gamma_{\alpha\beta}(\mathcal{A})]\} \\ &= W^{(3)}_{\mathrm{ChS}}[A] - (32\pi^2)^{-1} \int \mathrm{d}^3x\, \mathrm{tr}(\lambda^\alpha \lambda_\alpha) \end{aligned} \tag{28}$$

(in the Wess–Zumino gauge), where the last expression is nothing but the well-known $D = 3$ supersymmetric mass term for $\mathcal{A}_\alpha$ [12] (recall that $[\delta/\delta \mathcal{A}^{\alpha,a}]\, W^{\mathrm{SUSY}}_{\mathrm{ChS}}[\mathcal{A}] = -(8\pi^2)^{-1}\, \mathrm{tr}[T^a W_\alpha(\mathcal{A})]$). From (26), (28) and (12), (13) it follows that:

$$\eta^{\mathrm{SUSY}}_{\nabla^2}[\mathcal{A}^\omega] = \eta^{\mathrm{SUSY}}_{\nabla^2}[\mathcal{A}], \quad W^{\mathrm{SUSY}}_{\mathrm{ChS}}[\mathcal{A}^\omega] = W^{\mathrm{SUSY}}_{\mathrm{ChS}}[\mathcal{A}] + \mathcal{N}[\omega], \tag{29}$$

$$\eta^{\mathrm{SUSY}}_{\nabla^2}[\mathcal{A}^{\mathrm{P}}] = -\eta^{\mathrm{SUSY}}_{\nabla^2}[\mathcal{A}], \quad W^{\mathrm{SUSY}}_{\mathrm{ChS}}[\mathcal{A}^{\mathrm{P}}] = -W^{\mathrm{SUSY}}_{\mathrm{ChS}}[\mathcal{A}], \tag{30}$$

where $\mathcal{N}[\omega]$ is an integer and it may be viewed as superspace topological charge of $\omega(x,\theta)$ (12) [cf. (9)]:

$$\mathcal{N}[\omega] = -\mathrm{i}(48\pi^2)^{-1} \int \mathrm{d}^3x\, \mathrm{d}^2\theta\, \mathrm{tr}[(\omega^{-1}D^\alpha\omega)(\omega^{-1}D^\beta\omega)(\omega^{-1}\partial_{\alpha\beta}\omega)] = n_{D=3}[g], \tag{31}$$

i.e. only the bosonic component of $\omega(x,\theta)$ contributes. As a consequence of (29), (30) we have for $\mathcal{B}[\mathcal{A}]$ in (27):

$(\delta/\delta \mathcal{A}^{\alpha,a})\mathcal{B}[\mathcal{A}] = 0$ for all $\mathcal{A}_\alpha$ whose $\nabla^2(\mathcal{A})$ is invertible,

$$\mathcal{B}[\mathcal{A}^\omega] = \mathcal{B}[\mathcal{A}] - 2\mathcal{N}[\omega], \quad \mathcal{B}[\mathcal{A}^{\mathrm{P}}] = -\mathcal{B}[\mathcal{A}], \tag{32}$$

i.e. $\mathcal{B}[\mathcal{A}]$ is a piece-wise constant functional of $\mathcal{A}_\alpha$ whose values are even integers, just as in (10). In fact, comparing (4), (7)–(10) with (26)–(32) one easily finds:

$$\mathcal{B}[\mathscr{A}] = B[A], \tag{33}$$

i.e. only the zero modes of the ordinary Dirac operator $\nabla\!\!\!/(A)$, contained in $\nabla^2(\mathscr{A})$, contribute to the jumps (by ± 2) of $\mathcal{B}[\mathscr{A}]$.

Collecting (22)–(26) and assuming that there are $N_f \geqslant 1$ flavors [i.e. $\Phi_j(x,\theta), j = 1, ..., N_f$] we arrive at the final expression for (20):

$$N_f(\mathrm{sTr_R} \ln[\nabla^2])^{\mathrm{ren}} = N_f \tfrac{1}{2}(\mathrm{sTr_R} \ln[\nabla^4]) + \tfrac{1}{2} \mathrm{i}\pi N_f \eta_{\nabla^2}^{\mathrm{SUSY}}[\mathscr{A}] + N_f S_{\mathrm{ct}}[\mathscr{A}] \tag{34}$$

$$= N_f(\tfrac{1}{2}\mathrm{sTr_R} \ln[\nabla^4]) + \tfrac{1}{2}\mathrm{i}\pi N_f \mathcal{B}[\mathscr{A}] + \mathrm{i}\pi N_f W_{\mathrm{ChS}}^{\mathrm{SUSY}}[\mathscr{A}] + N_f S_{\mathrm{ct}}[\mathscr{A}]. \tag{34'}$$

Now we have the following two alternatives concerning PVA in (34):

(i) If either G = U(1) (then $\mathcal{N}[\omega] = n[g] = 0$) or if N_f = even for G = U(n), $n \geqslant 2$, we can choose: $S_{\mathrm{ct}}[\mathscr{A}] = -\mathrm{i}\pi W_{\mathrm{ChS}}^{\mathrm{SUSY}}[\mathscr{A}]$, i.e.

$$N_f(\mathrm{sTr_R} \ln[\nabla^2])^{\mathrm{ren}} = N_f(\tfrac{1}{2}\mathrm{sTr_R} \ln[\nabla^4]) + \tfrac{1}{2}\mathrm{i}\pi N_f \mathcal{B}[\mathscr{A}], \tag{35}$$

and thus PVA in (20), (34) are eliminated, since $\frac{1}{2}\pi N_f \mathcal{B}[\mathscr{A}] = 0 \pmod{2\pi}$.

(ii) If G = U(n), $n \geqslant 2$, and N_f = odd simultaneously, then the choice (35) while eliminating PVA, breaks gauge invariance [cf. (32)]. Hence PVAs are unavoidable in this case.

Conclusions (i), (ii) parallel those in the non-supersymmetric case [1,8].

In terms of component fields (11′–11″) in the Wess–Zumino gauge, eq. (34), accounting for (1), (4), (28), (33), reads:

$$N_f(\mathrm{Tr_R} \ln[\Delta_B] - \mathrm{Tr_R} \ln[-\mathrm{i}\nabla\!\!\!/])^{\mathrm{ren}} = N_f(\tfrac{1}{2}\mathrm{Tr_R} \ln[\Delta_B^2] - \tfrac{1}{2}\mathrm{i}\pi\eta_{\Delta_B}[A,\lambda])$$

$$- N_f(\tfrac{1}{2}\mathrm{Tr_R} \ln[\nabla\!\!\!/^2] - \tfrac{1}{2}\mathrm{i}\pi\eta_{\nabla\!\!\!/}[A]) + N_f S_{\mathrm{ct}}[A,\lambda], \tag{36}$$

$$\Delta_B \equiv \Delta_B[A,\lambda](x,x') = -\nabla_\mu(A)\nabla_\mu(A)\delta(x-x') - \tfrac{1}{4}\lambda^\alpha(x)[\nabla\!\!\!/(A)^{-1}]_\alpha{}^\beta(x,x')\lambda_\beta(x'),$$

$$\eta_{\Delta_B}[A,\lambda] = (16\pi^2)^{-1}\int \mathrm{d}^3x\, \mathrm{tr}(\lambda^\alpha\lambda_\alpha), \tag{36'}$$

or, in an equivalent form:

$$N_f(\mathrm{Tr_R} \ln[-\nabla_\mu(A)\nabla_\mu(A)] - \mathrm{Tr_R} \ln[-\mathrm{i}\Delta_F])^{\mathrm{ren}} = N_f \tfrac{1}{2}\mathrm{Tr_R} \ln[(-\nabla_\mu(A)\nabla_\mu(A))^2]$$

$$- N_f(\tfrac{1}{2}\mathrm{Tr_R} \ln[\Delta_F^2] - \tfrac{1}{2}\mathrm{i}\pi\eta_{\Delta_F}[A,\lambda]) + S_{\mathrm{ct}}[A,\lambda], \tag{37}$$

$$\Delta_F \equiv \Delta_F[A,\lambda](x,x') = \nabla\!\!\!/(A)_\alpha{}^\beta\delta(x-x') - \tfrac{1}{4}\lambda_\alpha(x)[-\nabla_\mu(A)\nabla_\mu(A)]^{-1}(x,x')\lambda^\beta(x'),$$

$$\eta_{\Delta_F}[A,\lambda] = \eta_{\nabla^2}^{\mathrm{SUSY}}[\mathscr{A}] = 2W_{\mathrm{ChS}}^{(3)}[A] + B[A] - (16\pi^2)^{-1}\int \mathrm{d}^3x\, \mathrm{tr}(\lambda^\alpha\lambda_\alpha). \tag{37'}$$

As a byproduct from (36), (37) one gets the η-invariants (36′), (37′) of the pseudodifferential (not ordinary differential) operators Δ_B, Δ_F in terms of the η-invariant (27) of the superdifferential operator $\nabla^2(\mathscr{A})$. On the other hand, direct computation of $\eta_{\Delta_B}, \eta_{\Delta_F}$ from expressions of the type (3) would be very hard.

Finally, let us stress the complete analogy among superspace formulas (34), (26)–(32) and the corresponding ordinary ones (1)–(4), (7)–(10). In particular, an application of supersymmetry to the computation of the spectral asymmetry of pseudodifferential operators is found.

We are very indebted to A.M. Polyakov and E.S. Sokatchev for many illuminating discussions.

References

[1] A.N. Redlich, Phys. Rev. D29 (1984) 2366;
R. Jackiw, in: Relativity, groups and topology II (Les Houches, 1983) eds. R. Stora and B.S. De Witt (North-Holland, Amsterdam, 1984).
[2] L. Alvarez-Gaumé and E. Witten, Nucl. Phys. B234 (1984) 269.
[3] A.J. Niemi and G.W. Semenoff, Phys. Rev. Lett. 51 (1983) 2077,
R. Jackiw, Phys. Rev. D29 (1984) 2375
[4] A.M. Polyakov, unpublished.
[5] J. Lott, Phys. Lett. 145B (1984) 179;
L. Alvarez-Gaumé, S. Della Pietra and G. Moore, Harvard preprint HUTP-84/A028 (1984);
Ya.I. Kogan and A.Yu. Morozov, ITEP preprints ITEP-104, 106 (1984).
[6] R. Hughes, Phys. Lett. 148B (1984) 215
[7] M Friedman, J. Sokoloff, Y. Srivastava and A. Widom, Phys. Rev Lett. 52 (1984) 1587;
K. Ishikawa, Phys. Rev. Lett. 53 (1984) 1615.
[8] E.R. Nissimov and S.J. Pacheva, Phys. Lett. 146B (1984) 227; in Proc. XIII Intern. Conf. on Differential geometric methods in theoretical physics (Shumen, Bulgaria, August 1984), eds. H. Doebner and T.D. Palev (World Scientific, Singapore), to be published;
E.S. Egorian, E.R. Nissimov and S.J. Pacheva, Sofia INRNE preprint (1984).
[9] M.F. Atiyah, V.K. Patodi and I.M. Singer, Math Proc. Camb Phil Soc. 77 (1975) 43, 78 (1975) 405; 79 (1976) 71
[10] B A. Dubrovin, S.P. Novikov and A.T. Fomenko, Modern geometry (Nauka, Moscow, 1979).
[11] S.S. Chern, Complex manifolds without potential theory (Springer, Berlin, 1979);
T Eguchi, P. Gilkey and A. Hanson, Phys. Rep. 66 (1980) 213.
[12] S J. Gates, M. Grisaru, M. Roček and W. Siegel, Superspace (Benjamin-Cummings, Reading, MA. 1983) Ch. I
[13] J. Honerkamp et al., Nucl. Phys. B95 (1975) 397;
I N. McArthur, Phys. Lett. 128B (1983) 194;
N.K. Nielsen, Nucl. Phys. B244 (1984) 499.
[14] F. Treves, Introduction to pseudodifferential and Fourier integral operators, Vol. 1 (Plenum, New York, 1982) Ch. III.